Problem 2.2

Show that the function

$$\psi(y,t) = (y - 4t)^2$$

is a solution of the differential wave equation. In what direction does it travel?

Solution

The aim is to show that the given function satisfies the wave equation,

$$\frac{\partial^2 \psi}{\partial t^2} \stackrel{?}{=} v^2 \frac{\partial^2 \psi}{\partial y^2},$$

with $v = 4$. Start taking derivatives, using the chain rule where appropriate.

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t}[(y - 4t)^2] = 2(y - 4t) \cdot (-4) = -8(y - 4t)$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial t}\right) = \frac{\partial}{\partial t}[-8(y - 4t)] = -8(-4) = 32$$

$$\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}[(y - 4t)^2] = 2(y - 4t) \cdot 1 = 2(y - 4t)$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial}{\partial y}[2(y - 4t)] = 2(1) = 2$$

Therefore,

$$\frac{\partial^2 \psi}{\partial t^2} = 32 = (16)(2) = 16\frac{\partial^2 \psi}{\partial y^2},$$

which means the given function for $\psi(y,t)$ is a solution of the wave equation.

$\psi(y,t) = (y - 4t)^2$ travels in the positive y-direction.